A Recurrent Latent Variable Model for Sequential Data

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Abstract

In this paper, we explore the inclusion of latent random variables into the dynamic hidden state of a recurrent neural network (RNN) by combining elements of the variational autoencoder. We argue that through the use of high-level latent random variables, our *variational RNN* (VRNN) is able to learn to model the kind of variability observed in highly-structured sequential data (such as speech). We empirically evaluate the proposed model against related sequential models on five sequential datasets, four of speech and one of handwriting. Our results show the importance of the role that latent random variables can play in the RNN dynamic hidden state.

1 Introduction

Learning generative models of sequences is a long-standing machine learning challenge and historically the domain of dynamic Bayesian networks (DBNs) such as hidden Markov models (HMMs) and Kalman filters. The dominance of DBN-based approaches has been recently overturned by a resurgence of interest in recurrent neural network (RNN) based approaches. An RNN is a special type of neural network that is able to handle both variable-length input and output. By training an RNN to predict the next output in a sequence, given all previous outputs, it can be used to model the joint probability distribution over sequences.

Both RNNs and DBNs consist of two parts: (1) a transition function that determines the evolution of the internal hidden state, and (2) a mapping from the state to the output. There are, however also a few important differences between RNNs and DBNs.

DBNs have typically been limited to either relatively simple state transition structures (e.g., linear models in the case of the Kalman filter) or to relatively simple internal state structure (e.g., the HMM state space consists of a single set of mutually exclusive states). RNNs, on the other hand, typically possess both a rich distributed internal state representation as well as flexible non-linear transition functions. These differences give RNNs extra expressive power in comparison to DBNs. This expressive power and the ability to train via error backpropagation are key reasons why RNNs have gained popularity as generative models for richly-structured sequence data.

In this paper we focus on another important difference between DBNs and RNN. While hidden state in DBNs is expressed in terms of random variables, the standard RNN state transition structure is entirely deterministic. The only source of randomness or variability in the RNN is found in the conditional output probability model. We suggest that this can be an inappropriate way to model of the kind of variability observed in highly-structured data, such as natural speech, which is characterized by strong and complex dependencies among the output variables at each timestep of the sequence.

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We argue, as have others [4, 2], that these complex dependencies cannot be modelled efficiently by standard RNN output models, which include either a simple unimodal distribution or a mixture of unimodal distributions.

We propose the use of high-level latent random variables to model the variability observed in the data. In the context of standard neural network models for non-sequential data, the recently introduced variational autoencoder (VAE) [11] offers an interesting combination of highly flexible non-linear mapping between the latent random state and the observed output and effective approximate inference. In this paper, we propose to extend the VAE into an recurrent framework for modelling high-dimensional sequences. The VAE can model complex multimodal distributions, which will help when the underlying true data distribution consists of multimodal conditional distributions. We call this joint model a *variational RNN*, or VRNN.

A natural question to ask is: how do we encode observed variability via latent random variables? The answer to this question depends on the nature of the data itself. In this work, we are mainly interested in highly-structured data that often arises in AI applications. By highly-structured, we mean that the data is characterized by two properties. First, there is a relatively high signal to noise ratio, meaning that the vast majority of the variability observed in the data is due to the signal itself and cannot reasonably be considered noise. Second, there exists a complex relationship between the underlying factors of variation and the observed data. For example, in speech, the vocal qualities of the speaker have a strong but complicated influence on the audio waveform, affecting the waveform in a consistent manner across frames.

With these considerations in mind, we suggest that our model variability should induce *dependencies across timesteps*. Thus, like DBN models such as HMMs and Kalman filters, we model dependencies between the latent random variables across timesteps. While we are not the first to propose to integrate random variables into the RNN hidden state, Boulanger-Lewandowski et al. [4], Bayer and Osendorfer [2], Fabius and van Amersfoort [6], we believe we are the first to integrate these dependencies between the latent random variables at neighboring timesteps.

We evaluate the proposed VRNN against other RNN-based models – including VRNNs without introducing temporal dependencies between the latent random variables – on two challenging sequential data types: natural speech and online handwriting. We demonstrate that for speech modelling the proposed VRNN significantly outperforms both RNNs and a similar model that does not integrate temporal dependencies between latent random variables.

2 Background

2.1 Sequence modelling with Recurrent Neural Networks

An RNN can take as input a variable-length sequence $x = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_T)$ by recursively processing each symbol while maintaining its internal hidden state $\mathbf{h}$. At each timestep t, an RNN reads the symbol $\mathbf{x}_t \in \mathbb{R}^d$ and updates its hidden state $\mathbf{h}_t \in \mathbb{R}^p$ by

$$\mathbf{h}_t = f_\theta\left(\mathbf{x}_t, \mathbf{h}_{t-1}\right), \tag{1}$$

where f is a deterministic non-linear transition function, and θ is its parameters. f can be implemented with gated activation functions such as long short-term memory [LSTM, 8] or gated recurrent unit [GRU, 5]. RNNs model sequences by parameterizing a factorization of the joint sequence probability distribution as a product of conditional probabilities such that

$$\begin{aligned} p(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_T) &= \prod_{t=1}^{T} p(\mathbf{x}_t \mid \mathbf{x}_{<t}), \\ p(\mathbf{x}_t \mid \mathbf{x}_{<t}) &= g_\tau(\mathbf{h}_{t-1}), \end{aligned} \tag{2}$$

where g is a function that maps the RNN state $\mathbf{h}_{t-1}$ to a probability distribution over possible outputs, and τ is the parameters of g.

One of the main factors which determines the representational power of an RNN is the output function g in Eq. (2). With a deterministic transition function f, the choice of g effectively defines the family of joint probability distributions $p(\mathbf{x}_1, \ldots, \mathbf{x}_t)$ that can be expressed by the RNN.

We can express the output function g in Eq. (2) as being composed of two parts. The first part φ_τ is a function that returns the parameter set ϕ_t given the hidden state $\mathbf{h}_{t-1}$, $\phi_t = \varphi_\tau(\mathbf{h}_{t-1})$, while the second part of g returns the density of $\mathbf{x}_t$, i.e. $p_{\phi_t}(\mathbf{x}_t \mid \mathbf{x}_{<t})$.

For the kind of real-valued and high-dimensional modelling tasks in which we are interested (i.e. naturak speech or handwriting), a reasonable choice of observation model is a Gaussian mixture model (GMM) as used in Graves [7]. In the case of GMM, φ_τ returns a set of mixture coefficients α_t, means $\boldsymbol{\mu}_{\cdot,t}$ and covariances $\Sigma_{\cdot,t}$ of the corresponding mixture components. The probability of $\mathbf{x}_t$ under the mixture distribution is

$$p_{\boldsymbol{\alpha}_t,\boldsymbol{\mu}_{\cdot,t},\Sigma_{\cdot,t}}(\mathbf{x}_t|\mathbf{x}_{<t}) = \sum_j \alpha_{j,t}\mathcal{N}\left(\mathbf{x}_t; \boldsymbol{\mu}_{j,t}, \Sigma_{j,t}\right).$$

With the notable exception of [7], there have been few works investigating the structured output density model for RNNs with real-valued sequences.

There is potentially a significant issue in the way the RNN models output variability. Given a deterministic transition function, the only source of variability is in the conditional output probability density. This can present problems when modelling sequences that are at once highly variable and highly structured. To effectively capture this kind of data, the RNN must be capable of mapping very small variations in $\mathbf{x}$ (the only source of randomness) to potentially very large variations in the hidden state $\mathbf{h}_t$. Limiting the capacity of the network (as must be done to guard against overfitting) will force a compromise between the generation of a clean signal (i.e. with high signal to noise ratio) and encoding sufficient input variability to capture the high-level variability both within a single observed sequence and across data examples.

The need for highly-structured output functions in an RNN has been previously noted. Boulanger-Lewandowski et al. [4] extensively tested NADE and RBM-based output densities for modelling sequences of binary vector representations of music. Bayer and Osendorfer [2] introduced a sequence of independent latent variables corresponding to the states of the RNN. Their model, called *STORN*, first generates a sample $(\mathbf{z}_1, \ldots, \mathbf{z}_T)$ from the sequence of independent latent random variables. At each timestep, the transition function f from Eq. (1) computes the next hidden state $\mathbf{h}_t$ based on both the previous state $\mathbf{h}_{t-1}$, the previous output $\mathbf{x}_{t-1}$ as well as the sampled latent random variable $\mathbf{z}_t$. They proposed to train this model based on the VAE principle (see Sec. 2.2.) Similarly, Pachitariu and Sahani [14] earlier proposed both a sequence of independent latent random variables and a stochastic hidden state for the RNN.

These approaches are closely related to the approach proposed in this paper. However, there is a major difference in how the prior distribution over the latent random variable $\mathbf{z}$ is modelled. Unlike them, our approach makes the prior distribution of the latent random variable at time t dependent on all the preceding inputs via the RNN hidden state $\mathbf{h}_{t-1}$ (see Eq. (5)). The introduction of temporal structure into the prior distribution is expected to improve the representational power of the model, which we empirically observe in the experiments (See Table 1). We call this techniuqe as using a sequential prior. However, it is important to note that any approach based on having stochastic latent states is orthogonal to having a structured output function, and these two can be used together to form a single model.

2.2 Variational Autoencoder

For non-sequential data, VAEs [11, 15] have recently been shown to be an effective modelling paradigm to recover complex multimodal distributions over the data space.

A VAE introduces a set of latent random variables $\mathbf{z}$, designed to capture the variations in the observed variables $\mathbf{x}$. As an example of a directed graphical model, the joint distribution is defined as

$$p(\mathbf{x}, \mathbf{z}) = p(\mathbf{x} \mid \mathbf{z})p(\mathbf{z}). \tag{3}$$

where the prior over the latent random variables, $p(\mathbf{z})$, is generally chosen to be a simple Gaussian distribution and the conditional $p(\mathbf{x} \mid \mathbf{z})$ is an arbitrary observation model whose parameters are computed by a parametric function of $\mathbf{z}$. Importantly, the VAE typically parameterizes $p(\mathbf{x} \mid \mathbf{z})$ with a highly flexible function approximator such as a neural network. While latent random variable models of the form given in Eq. 3 are not uncommon, endowing the conditional $p(\mathbf{x} \mid \mathbf{z})$ as a potentially highly non-linear mapping from $\mathbf{z}$ to $\mathbf{x}$ is a rather unique feature of the VAE.

However, introducing a highly non-linear mapping from $\mathbf{z}$ to $\mathbf{x}$ results in intractable inference of the posterior $p(\mathbf{z} \mid \mathbf{x})$. Instead, the VAE uses a variational approximation $q(\mathbf{z} \mid \mathbf{x})$ of the posterior that enables the use of the lower bound

$$\log p(\mathbf{x}) \geq -\mathrm{KL}(q(\mathbf{z} \mid \mathbf{x}) \| p(\mathbf{z})) + \mathbb{E}_{q(\mathbf{z}|\mathbf{x})}\left[\log(p(\mathbf{x} \mid \mathbf{z}))\right], \tag{4}$$

where $\mathrm{KL}(Q\|P)$ is Kullback-Leibler divergence between two distributions Q and P.

In Kingma and Welling [11], the approximate posterior $q(\mathbf{z} \mid \mathbf{x})$ is a Gaussian $\mathcal{N}(\boldsymbol{\mu}, \mathrm{diag}(\boldsymbol{\sigma}^2))$ whose mean $\boldsymbol{\mu}$ and variance $\boldsymbol{\sigma}^2$ are the output of a highly non-linear function of $\mathbf{x}$, once again typically a neural network.

The generative model $p(\mathbf{x} \mid \mathbf{z})$ and inference model $q(\mathbf{z} \mid \mathbf{x})$ are then trained jointly by maximizing the variational lower bound with respect to their parameters, where the integral with respect to $q(\mathbf{z} \mid \mathbf{x})$ is approximated stochastically. The gradient of this estimate can have a low variance estimate, by reparametrizing $z = \boldsymbol{\mu} + \boldsymbol{\sigma} \odot \boldsymbol{\epsilon}$ where $\boldsymbol{\epsilon}$ is a standard Gaussian variable and rewriting

$$\mathbb{E}_{q(\mathbf{z}|\mathbf{x})}\left[\log p(\mathbf{x} \mid \mathbf{z})\right] = \mathbb{E}_{p(\boldsymbol{\epsilon})}\left[\log p(\mathbf{x} \mid \mathbf{z} = \boldsymbol{\mu} + \boldsymbol{\sigma} \odot \boldsymbol{\epsilon})\right].$$

The inference model can then be trained through standard backpropagation technique for stochastic gradient descent.

3 Variational Recurrent Neural Network

In this section, we introduce a recurrent version of the VAE for the purpose of modelling sequences. Drawing inspiration from simpler dynamic Bayesian networks (DBNs) such as HMMs and Kalman filters, the proposed variational recurrent neural network (VRNN) explicitly models the dependency between latent random variables across subsequent timesteps. However, unlike these simpler DBN models, the VRNN retains the flexibility to model highly non-linear dynamics.

Generation The VRNN contains a VAE at every timestep. However, these VAEs are conditioned on the state variable $\mathbf{h}_{t-1}$ of an RNN. This addition will help us to take into account the temporal structure of the sequential data. Unlike a standard VAE, the prior on the latent random variable is no longer a standard Gaussian distribution, but follows the distribution

$$\mathbf{z}_t \sim \mathcal{N}(\boldsymbol{\mu}_{0,t}, \mathrm{diag}(\boldsymbol{\sigma}_{0,t}^2)), \text{ where } [\boldsymbol{\mu}_{0,t}, \boldsymbol{\sigma}_{0,t}] = \varphi_\tau^{\text{prior}}(\mathbf{h}_{t-1}), \tag{5}$$

where $\boldsymbol{\mu}_{0,t}$ and $\boldsymbol{\sigma}_{0,t}$ denote the parameter set of the conditional prior distribution, and we call it as a sequential prior. Moreover, the generating distribution will not only be conditioned on $\mathbf{z}_t$ but also on $\mathbf{h}_{t-1}$ to be

$$\mathbf{x}_t \mid \mathbf{z}_t \sim \mathcal{N}(\boldsymbol{\mu}_{x,t}, \mathrm{diag}(\boldsymbol{\sigma}_{x,t}^2)), \text{ where } [\boldsymbol{\mu}_{x,t}, \boldsymbol{\sigma}_{x,t}] = \varphi_\tau^{\text{dec}}(\varphi_\tau^{\mathbf{z}}(\mathbf{z}_t), \mathbf{h}_{t-1}), \tag{6}$$

where $\boldsymbol{\mu}_{x,t}$ and $\boldsymbol{\sigma}_{x,t}$ denote the parameter set of the generating distribution, $\varphi_\tau^{\text{prior}}$ and $\varphi_\tau^{\text{dec}}$ can be any highly flexible function such as neural networks. $\varphi_\tau^{\mathbf{x}}$ and $\varphi_\tau^{\mathbf{z}}$ can also be neural networks, that will extract complex features from $\mathbf{x}_t$ and $\mathbf{z}_t$, respectively. We found that these feature extractors are crucial for enabling the learning of complex sequences for the model.

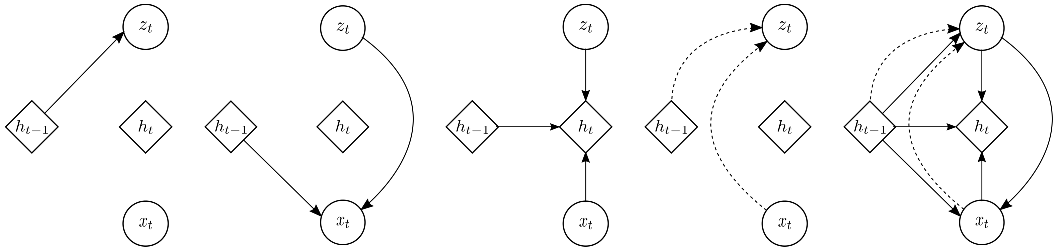
The RNN will update its internal hidden state using the recurrence equation:

$$\mathbf{h}_t = f_\theta\left(\varphi_\tau^{\mathbf{x}}(\mathbf{x}_t), \varphi_\tau^{\mathbf{z}}(\mathbf{z}_t), \mathbf{h}_{t-1}\right), \tag{7}$$

where f was originally the transition function from Eq. (1).

This parameterization of the generative model results in and – was motivated by – the factorization

$$p(\mathbf{x}_{\leq T}, \mathbf{z}_{\leq T}) = \prod_{t=1}^{T} p(\mathbf{x}_t \mid \mathbf{z}_{\leq t}, \mathbf{x}_{<t}) p(\mathbf{z}_t \mid \mathbf{x}_{<t}, \mathbf{z}_{<t}). \tag{8}$$



(a) Prior (b) Generation (c) Recurrence (d) Inference (e) VRNN

Figure 1: Graphical illustrations of each operation in the proposed VRNN: (a) computing sequential priors using Eq. (5); (b) generating function using Eq. (6); (c) updating the recurrence of the RNN part using Eq. (7); (d) inference of the approximate posterior using Eq. (9); (e) overall computational paths of the VRNN.

Inference In a similar fashion, the approximate posterior will not only be a function of $\mathbf{x}_t$ but also of $\mathbf{h}_{t-1}$ following the equation:

$$\mathbf{z}_t \mid \mathbf{x}_t \sim \mathcal{N}(\boldsymbol{\mu}_{z,t}, \text{diag}(\boldsymbol{\sigma}_{z,t}^2)) \text{ , where } [\boldsymbol{\mu}_{z,t}, \boldsymbol{\sigma}_{z,t}] = \varphi_\tau^{\text{enc}}(\varphi_\tau^{\mathbf{x}}(\mathbf{x}_t), \mathbf{h}_{t-1}), \tag{9}$$

similarly $\boldsymbol{\mu}_{z,t}$ and $\boldsymbol{\sigma}_{z,t}$ denote the parameter set of the approximate posterior. We can notice that the encoding of the approximate posterior and the decoding for generation are also tied through the hidden state $\mathbf{h}_{t-1}$. We can also observe that this results in the factorization

$$q(\mathbf{z}_{\leq T} \mid \mathbf{x}_{\leq T}) = \prod_{t=1}^{T} q(\mathbf{z}_t \mid \mathbf{x}_{\leq t}, \mathbf{z}_{<t}). \tag{10}$$

Learning This factorization is crucial in breaking the variational lower bound into timestep-wise

$$\int \log \left(\frac{p(\mathbf{x}_{\leq T}, \mathbf{z}_{\leq T})}{q(\mathbf{z}_{\leq T} \mid \mathbf{x}_{\leq T})} \right) dq(\mathbf{z}_{\leq T} \mid \mathbf{x}_{\leq T}) = \sum_{t=1}^{T} -\text{KL}(q(\mathbf{z}_t \mid \mathbf{x}_{\leq t}, \mathbf{z}_{<t}) \| p(\mathbf{z}_t \mid \mathbf{x}_{<t}, \mathbf{z}_{<t})) + \mathbb{E}_{q(\mathbf{z}_t \mid \mathbf{x}_{\leq t}, \mathbf{z}_{<t})} \left[\log(p(\mathbf{x}_t \mid \mathbf{z}_{\leq t}, \mathbf{x}_{<t})) \right].$$

As in the standard VAE, we learn the generative and inference models jointly by maximizing the variational lower bound with respect to their parameters. The schematic view of the VRNN is shown in Fig. 1, each of (a)–(d) operation corresponds to each of Eqs. (5),(6),(7),(9). The proposed network applies the operation (a), hence, it has a sequential prior (VRNN, see Eq. (5)). The variant of the VRNN which does not apply the operation (a), then the prior is independent across timesteps (VRNN-I). STORN [2] model can be considered an instance of the VRNN-I model family. In fact, STORN makes further restrictions on the dependency structure of the approximate inference model. We include this version of the model (VRNN-I) in our experimental evaluation in order to directly study the impact of including the temporal dependency structure in the prior (sequential prior) over the latent random variables.

4 Experiment Settings

We evaluate the proposed VRNN model on two tasks: (1) modelling natural speech directly from the raw audio waveform; (2) modelling the dynamic handwriting process.

Speech modelling We train the models to directly model raw audio, represented as a sequence of 200-dimensional frames. Each frame corresponds to the real-valued amplitudes of 200 consecutive raw acoustic samples. Note that this is unlike the conventional approach to modelling speech, often used in speech synthesis where models are expressed over representations such as spectral features [see, e.g., 16, 3, 12].

We evaluate the models on the following four speech datasets:

Table 1: Average log-probability on the test (or validation) set of each task.

Models	Speech modelling				Handwriting
	Blizzard	TIMIT	Onomatopoeia	Accent	IAM-OnDB
RNN-Gauss	3539	-1900	-984	-1293	1016
RNN-GMM	7413	26643	18865	3453	1358
VRNN-I-Gauss	≥ 8933	≥ 28340	≥ 19053	≥ 3843	≥ 1332
	≈ 9188	≈ 29639	≈ 19638	≈ 4180	≈ 1353
VRNN-Gauss	≥ 9223	≥ 28805	≥ 20721	≥ 3952	≥ 1337
	$\approx \mathbf{9516}$	$\approx \mathbf{30235}$	$\approx \mathbf{21332}$	≈ 4223	≈ 1354
VRNN-GMM	≥ 9107	≥ 28982	≥ 20849	≥ 4140	≥ 1384
	≈ 9392	≈ 29604	≈ 21219	$\approx \mathbf{4319}$	$\approx \mathbf{1384}$

1. **Blizzard**: This text-to-speech dataset made available by the Blizzard Challenge 2013 contains 300 hours of English spoken by a single female speaker [9].
2. **TIMIT**: This most widely used datasets for benchmarking speech recognition systems contains $6,300$ English sentences ready by 630 speakers.
3. **Onomatopoeia**[1]: Onomatopoeia is a set of $6,738$ non-linguistic human-made sounds such as coughing, screaming, laughing and shouting, recorded from 51 voice actors.
4. **Accent**: This dataset contains English paragraphs read by $2,046$ different native and non-native English speakers [17].

For the Blizzard and Accent datasets, we process the data so that each sample duration is $0.5s$ (the sampling frequency used is 16kHz). We use *truncated backpropagation through time* and initialize the hidden state of the RNN part with the final hidden state of previous minibatch, resetting to a zero-vector every four updates. Excluding TIMIT, the rest of the datasets do not have predefined train/test splits. We shuffle and divide the data into train/validation/test splits using a fraction of $0.9/0.05/0.05$. See supplementary material for more details on processing datasets and experimental settings.

Handwriting Generation We let each model learn a sequence of (x, y)-coordinates together with binary indicators of pen up / pen down, using the IAM-OnDB dataset which consists of $13,040$ handwritten lines written by 500 writers [13]. We preprocess and split the dataset as done in [7].

Preprocessing and Training The only preprocessing used in the experiments is normalizing each vector of a sequence by using the global mean and standard deviation computed from the training set. We trained each model by the stochastic gradient descent on the negative log-likelihood using the recently proposed Adam optimizer [10], with learning rate of 0.001 for TIMIT and Accent, 0.0003 for the rest. We used minibatch size of 128 for Blizzard and Accent, and 64 for the rest. The final model was chosen with early-stopping the training based on the validation performance.

Models We compare the proposed VRNN with a standard RNN. For each architecture, we evaluate two different output functions: unimodal Gaussian distribution (Gauss) and the Gaussian mixture model (GMM). For each task, we conduct additional set of experiments of VRNN without sequential prior (VRNN-I).

We fix the size of the RNN of each model to have single recurrent hidden layer with 2000 LSTM units (in the case of Blizzard, 4000 and for IAM-OnDB, 1200). All the φ_τ in Eqs. (5)–(9) have four hidden layers using rectified linear units (for IAM-OnDB, we use single hidden layer).

The standard RNN models (which begin with RNN-) only have $\varphi_\tau^{\mathbf{x}}$ and $\varphi_\tau^{\text{dec}}$, while the proposed VRNN models have $\varphi_\tau^{\mathbf{z}}$, $\varphi_\tau^{\text{enc}}$ and $\varphi_\tau^{\text{prior}}$ as well. For standard RNNs, $\varphi_\tau^{\mathbf{x}}$ is the feature extractor, and $\varphi_\tau^{\text{dec}}$ is the generating function. The standard RNN means that it uses Eq.(1) to update its internal hidden state. For the RNN-GMM and VRNNs, we match the number of parameters of each output function as closely as possible to an output function of an RNN-Gauss model having 600 rectified

[1] This dataset has been provided by Ubisoft.

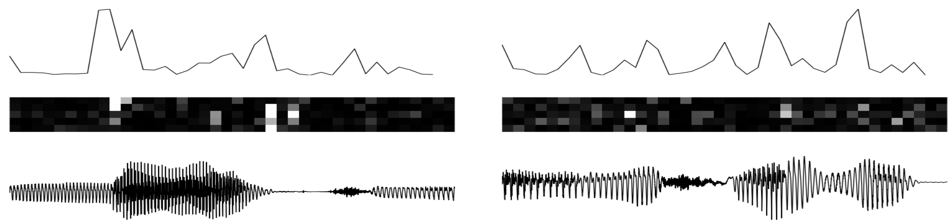


Figure 2: The top row represents the difference δ_t between $\boldsymbol{\mu}_{z,t}$ and $\boldsymbol{\mu}_{z,t-1}$. The middle row represents the dominant KL divergence values shown in temporal order. The bottom row shows corresponding waveforms.

linear units for any hidden layer that belongs to either $\varphi_\tau^{\mathbf{x}}$ or $\varphi_\tau^{\text{dec}}$ (800 for Blizzard). Note that the models using GMM (RNN-GMM & VRNN-GMM) have 20 mixture components.

For qualitative analysis for speech, we train larger models to generate sequences, but again control the number of parameters. For all models, we use stacked RNNs with three recurrent hidden layers, each layer contains 3000 LSTM units. For the non-RNN parts, we match the number of parameters for the output function to an output function of RNN-Gauss model having 3200 rectified linear units for all the hidden layers that belong to $\varphi_\tau^{\mathbf{x}}$ and $\varphi_\tau^{\text{dec}}$.

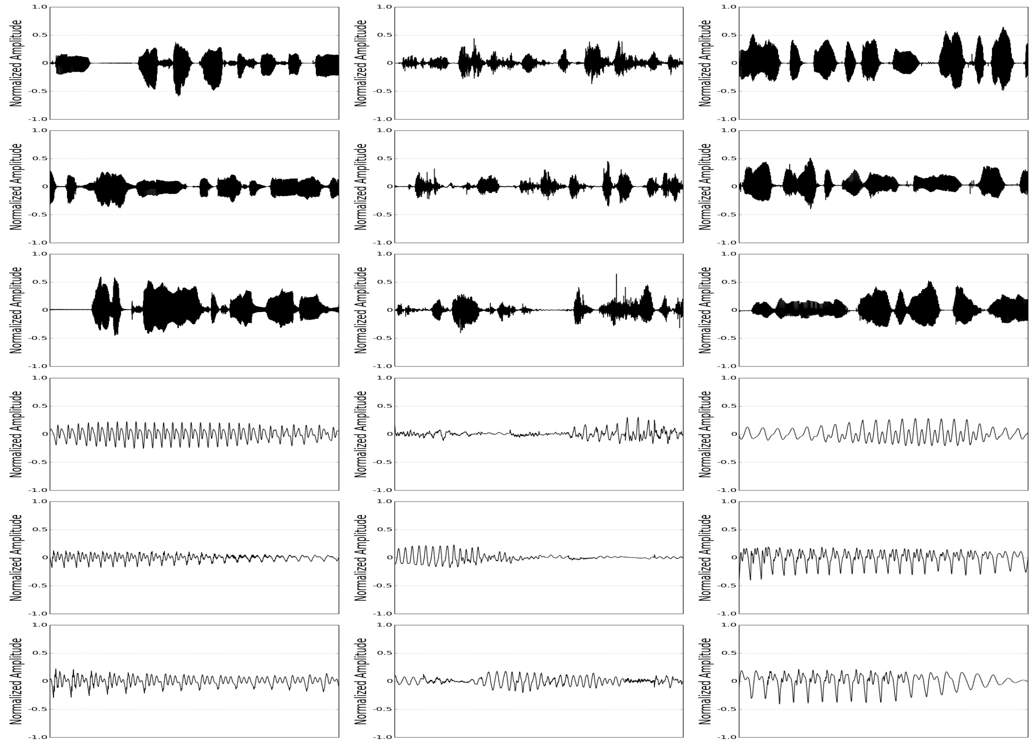
5 Results and Analysis

We evaluate the average log-probability of test examples assigned by each model and report in Table 1. With RNN-Gauss and RNN-GMM, we report exact log-probabilities, while in the case of VRNNs we report the variational lower bound (given with $\geq$ sign, see Eq. (4)) and approximated marginal log-likelihood (given with $\approx$ sign) based on importance sampling using 40 samples as in Rezende et al. [15]. In all cases, higher scores are better. Our results show the proposed VRNNs have better log-probability performances which support our claim that latent random variables are helpful when modelling comlex sequences. VRNN-Gauss performs well (compared to VRNN-GMM) using only an unimodal output function, which does not happen in the standard RNN case.

Latent Space Analysis After observing the improvements achieved by the proposed VRNN, we were curious on what kind of dynamics of the latent random variables have been learned by the model. In Fig. 2, we show analysis of the latent random variables. We let the VRNN to read some unseen examples and observe changes in the states of latent random variables. We compute $\delta_t = \sum_j (\boldsymbol{\mu}_{z,t}^j - \boldsymbol{\mu}_{z,t-1}^j)^2$ at every timestep t and plot the results on the topmost row of Fig. 2. We can clearly observe the peaks of δ_t, whenever there is a transition in the waveform (shown at the bottom row), reflecting the changes of modality in the RNN dynamics. The middle row shows the KL divergence computed between the approximate posterior and the sequential prior. When there is a transition, the KL divergence tends to grow (white is high).

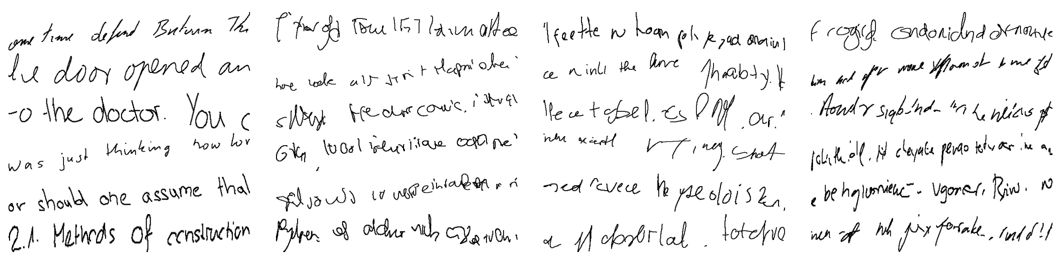
Speech Generation We generate waveforms with $2.0s$ duration from the models that were trained on Blizzard. From Fig. 3, we can clearly see that the waveforms from the VRNN-Gauss are much less noisy and have less spurious peaks than those from the RNN-GMM. We suggest that the small amount of noise apparent in the RNN-GMM model is a consequence of the compromise these models must make between representing a clean signal consistently to the training data and encoding sufficient input variability to capture the variations across data examples. The latent random variable models (both VRNN-I and VRNN) can avoid this compromise by adding variability in the latent space which can always be mapped to a point close to a relatively clean sample.

Handwriting Generation Visual inspection of generated handwriting from the trained models reveals that the proposed VRNN tends to generate with more diverse and consistent writing styles, when compared to the RNN-GMM. Fig 4 depicts handwriting from the training examples, RNN-Gauss, RNN-GMM and VRNN-GMM.



(a) Ground Truth (b) RNN-GMM (c) VRNN-Gauss

Figure 3: Typical training examples and generated samples from RNN-GMM and VRNN-Gauss. Top three rows show the global waveforms while the bottom three rows show more zoomed-in waveforms. Samples from (b) RNN-GMM contain high frequency noise, (c) VRNN-Gauss generates samples with less noise. We excluded RNN-Gauss because the samples are almost close to pure noise.



(a) Ground Truth (b) RNN-Gauss (c) RNN-GMM (d) VRNN-GMM

Figure 4: Handwriting samples: (a) ground truth examples from the training examples; unconditionally generated handwritings from (b) RNN-Gauss, (c) RNN-GMM and (d) VRNN-GMM. The VRNN-GMM retains writing styles from beginning to end while RNN-Gauss and RNN-GMM tend to change style during the generation process. This is possibly because sequential latent random variables guide the model to generate samples with a consistent writing style.

6 Conclusion

We propose a novel model of complex sequential data that incorporates latent random variables into a recurrent neural network (RNN) architecture. We show that by modelling the dependencies between these latent random variables, we are able to provide a model that naturally reflects the kinds of variability seen in many sequential processes.

Our experiments focus on unconditional speech generation involving various real-valued datasets as well as unconstrained handwriting generation. We find the introduction of latent random variables provides a significant performance increase for unconditional speech modelling and further show the importance of temporal conditioning of these latent random variables. Samples from VRNN models are qualitatively competitive with existing methods, and appear to show stylistic consistency over the course of generation. This is especially apparent in handwriting samples.

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